

# C.9.\_Ulfa\_Wijaya\_2019\_Expanding\_HLT\_in\_Math\_Inst.pdf

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**Submission date:** 23-Apr-2020 04:42PM (UTC+0700)

**Submission ID:** 1305408127

**File name:** C.9.\_Ulfa\_Wijaya\_2019\_Expanding\_HLT\_in\_Math\_Inst.pdf (607.16K)

**Word count:** 3398

**Character count:** 18678

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To cite this article: Charina Ulfa and Ariyadi Wijaya 2019 *J. Phys.: Conf. Ser.* **1320** 012091

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## Expanding hypothetical learning trajectory in mathematics instructional

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**Abstract.** The purpose of this article is to present a description of Hypothetical Learning Trajectory (HLT) which can be used as an instrument in mathematics learning activities in the classroom. More than 25 years, HLT has been introduced and developed in every learning especially mathematics learning. An HLT is developed based on the learning objectives, characteristics of students and the material to be taught. In HLT contains three important things that are: (1) learning objectives, (2) learning activities used in building students' knowledge, and (3) allegations of the learning activities process. This article uses a literature review of previous theories. HLT can be developed with various learning methods that are adjusted to the characteristics of students and the material to be taught. However, this article only discusses important things that need to be considered in the development of HLT in mathematics learning. So that it is obtained that HLT can help students in building knowledge in a reflective manner.

### 1. Introduction

Considering that mathematics is an abstract and complex science for students in learning, therefore mathematics learning always developing to help students for reach mathematical knowledge. The main purpose of mathematics education is to support students' skills in mathematics in various life situations [1]. As we have seen, that everyday life has never escaped the use of mathematics. Therefore, it can be said that mathematics is a science used in sharing knowledge. This is in accordance with one of the principles of NCTM which says that mathematics is the key of opportunities to get the other knowledge [2, 3].

In mathematics education, there is a learning process that needs more attention than learning outcomes. The learning process is a learning activity that should be planned maximally. Furthermore, students will be interested in learning process, if the learning process is designed and planned properly [4]. If the mathematics learning process is well designed, good learning outcomes will also be obtained. The same thing was also conveyed by Brousseau who said that students will not learn anything if the educator (teachers or researchers) does not have the intentions, plans, and learning situations that are well organized [5]. Hypothetical learning trajectory (HLT) is one instrument that can develop the learning process. The development of learning trajectories is to describe the important aspects that are the key to planning mathematics learning [5].

Hypothetical learning trajectory (HLT) or also called the alleged learning trajectory has been developed for more than 25 years. Baker said that hypothetical learning trajectory (HLT) is a link between a learning theory and actual learning practices [6]. In accordance with the statement that said



that HLT specifically describes how educators design mathematics learning and choose math assignments that display conceptual learning of mathematics [7]. Furthermore Simon also said that HLT as an illustration of the learning process where students build knowledge by conducting a series of learning activities to achieve learning goals [8].

HLT aims to produce a learning tool that helps students to gain knowledge that will be long to remember. HLT is a picture of students' thoughts in the form of allegations or hypotheses from the design of a series of learning activities that will encourage students to reach the expected learning goals [9]. Learning trajectory is also very supportive for an assumption about the level or direction of knowledge, which students will go through when learning mathematics [10]. The main idea of HLT was to give a unique and different contribution because it involved the formation of reflective (thinking) knowledge for students. Reflective knowledge is knowledge gained by students as a result of intellectual and affective activities that explore with more experiences to get new knowledge [11]. Not only reflective knowledge, but HLT also affects the interaction process in learning [12].

This article aims to explain things that need to be considered in developing HLT in mathematics learning with the aim of obtaining instructional theory that can be used as one of the solutions to educational problems. By knowing this, teachers and researchers can make a good learning path of mathematics by considering the direction of thinking of students. So that later it will produce a learning trajectory that greatly supports students in learning mathematics.

## 2. Research Method

The research method in this study uses literature review of several articles both from Indonesia and outside Indonesia. The articles were reviewed to get the important things needed to discuss HLT. Starting with what needs to be considered before developing HLT and how to develop HLT in mathematics learning?

In each paper studied, we take several important points that can be used as material to support and develop this article. We also compare the things that are considered important for further comparison with the results of research obtained from other articles. The articles chosen to be used as material in writing this article are articles that contain HLT and design research.

## 3. Discussion

### 3.1. Learning trajectory

Learning trajectory was first introduced by Simon, which made HLT in a learning theory that became one of the ways to present pedagogical thinking on mathematical understanding that involves teaching mathematics. HLT is very helpful in connecting learning theory with learning practices by constructing learning content to support students in achieving the objectives of learning mathematics [13]. According to Simon, HLT is still said to be a notion of the learning trajectory because the actual learning trajectory was unknown yet [14]. HLT is a teaching construction designed by educators as a way to understand students and begin conceptual learning in mathematics [7], [14]. Furthermore HLT is a learning trajectory that provides a general description of the learning process of students [15].

Simon also believes that Learning Trajectory (LT) brings more integration of students' mathematical experiences by connecting between ways in developing students' mathematical concepts from informal understanding to formal mathematical concepts [16]. Hypothetical Learning Trajectory (HLT) is still an alleged flow of learning towards a set of mathematical learning activities that students will go through in getting a mathematical knowledge. While Learning Trajectory (LT) is a learning flow in a series of mathematics learning activities that students have passed after testing and improvement of HLT, resulting in an actual flow and more efficient in helping students to gain new knowledge. It can be said that learning trajectory developed through empirical research designed to identify the steps of activities that students follow when they develop their initial mathematical ideas into formal concepts [8].

Designing HLT is also not an easy thing to be done by educators, it needs deepening of the material and characteristics of students. This is appropriate with Simon's statement [16], [17] which says that designing HLT which is to reform mathematics is not an easy task. Therefore, before designing HLT, there are four aspects that are interrelated and important to know: (1) learning objectives about students' reasoning; (2) sequence of instructional tasks (activities) in which students are involved; (3) changes in students' mathematical activities; and (4) the role of educators in supporting the mathematical development of students [13]. Simon and Tzur also define HLT by using four principles which are almost with statement before, that is: (1) HLT generation is based on understanding current knowledge of the students involved, (2) HLT is a tool for planning learning with certain mathematical concepts, (3) the task of mathematics provides learning of certain mathematical concepts so that the task becomes a key part of the learning process, and (4) because it is still hypothetical, so that educators are involved in modifying every aspect of HLT. Therefore, educators (teachers) play an active role in the development of HLT [17].

HLT is still in the form of a theoretical study that still needs to be researched and experimented in small groups, then which can be tested on larger groups such as in classroom. HLT can also be said as an instrument that combines the theory of instruction (learning) and practice (experiment) teaching in design research [18] There are three parts in (learning trajectory), that is: mathematical goals, development paths where students can develop to achieve learning goals, and a set of activities or mathematical tasks, which are matched with each level of students' thinking so that can help students to develop thinking skills [19] This is similarly conveyed by Simon that he defines that HLT consists of three components: (1) directed learning objectives, (2) learning activities, and (3) learning processes that can predict how the students' think and students' understanding can be develop with the process of learning activities [18].

Before designing HLT, the educator or reseracher must know and capable in the material to be taught and know how the students are able to form an understanding of mathematical concepts that start from non-formal forms into a form of formal mathematical concepts[5]. To find out this, researchers can create a design activity and learning trajectory that can help students. In achieving a learning goal, as an educator, the teacher is the main source in making and developing learning trajectories in accordance with Figure 1. Therefore in developing HLT requires knowledge and mastery of educators who are good at the material to be taught.

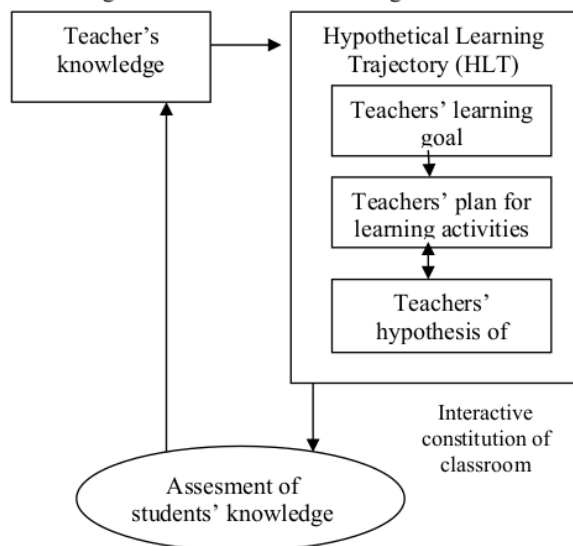


Figure 1. Mathematics learning cycle [5]



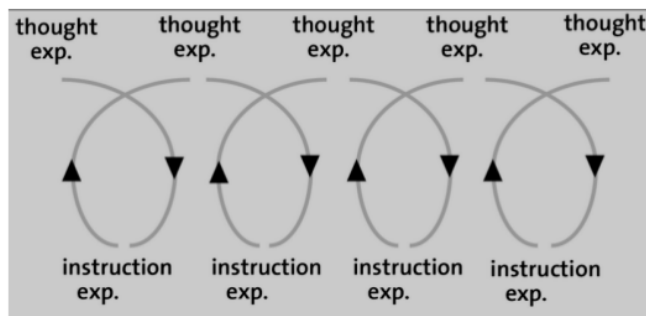
HLT explains and describes the path that students should take in learning a mathematical material or new mathematical knowledge [13], [7]. For this reason, researchers who will develop HLT will be better of working with the teacher to obtain HLT that will help students learn mathematics. So that, there are three parts that become priorities in developing HLT, those are:

- 1) Learning objectives: developing learning activities must be adapted to the purpose of learning mathematics, so that learning objectives should be arranged according to the characteristics of the material to be studied.
- 2) Learning activities: design and compile a series of activities that are directed and adapted to the learning objectives that are arranged, so that students get mathematical knowledge based on the activities that have been conducted.

Estimate (hypotesis) of learning trajectories (ways) : make an alleged learning flow based on planned activities, which students will go through in developing their knowledge in mathematics learning

### 3.2. Using design research in HLT

Every HLT that has been designed, then that designed will be tested (experiment) in mathematics learning. This HLT research uses design research which is the relationship between a learning theory and practicing mathematics. Design research in education aims to develop a solution to complex problems in educational practice and to develop or validate a learning theory [20]. In the figure below is presented the schema of design research methods by Gravemeijer and Cobb which get the new theory of instructional [21]. By using the design research, we need at least 2 cycles to get the new theory of instructional.



**Figure 2.** Design Research cycle [21]

This design research aims to produce a new learning theory or Local Instructional Theory (LIT), with the instrument used in this design research is the HLT design itself. Liljekvist said that who use this design research aims to understand and know more about the relationship between teaching and learning in order to improve teaching [22]. Furthermore Barab and Squire also added that design research is an approach that aims to produce a new theory in learning practices that are able to influence the learning and teaching processes naturally (naturally) [21].

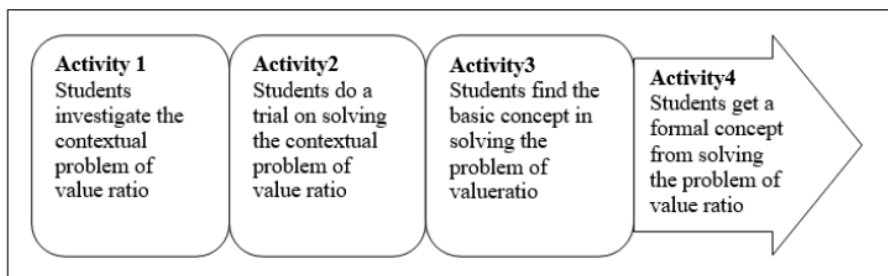
Design research can be defined by differences in objectives that distinguish between development studies and validation studies [20]. The development study is intended as a systematic analysis, design and evaluation of educational interventions with the dual purpose of producing research-based solutions for complex problems in learning practices and advancing knowledge of the characteristics of interventions and their design and development processes. While validation studies are intended as intervention studies (such as learning processes, learning environments) with the aim of developing or validating theories about the process and how the process can be designed [23]. There is no specific opinion that says the criteria of the success rate of HLT have been prepared, because HLT is still an alleged flow of learning. In allegations there are no criteria or categories that measure success or not,

just by achieving the learning objectives that have been compiled, the alleged learning flow has been reached. HLT tests in stages and undergoes a revision (improvement) of an assumption that the goal is not achieved until the achievement of a learning trajectory or often known as local instructional theory. There is no specific standard in determining the success of HLT, because HLT itself is still an alleged learning flow. HLT achievement depends on the achievement of planned learning goals.

### 3.3. Developing HLT in mathematics learning

In developing HLT, the teacher or researcher has thought about a path plan that students will go through in reaching new mathematical knowledge. In this case, the teacher or researcher is required for professionals both in mastering the material and students [24], [25]. The next step the teacher makes the design of learning that will be through by students, where is in the beginning of learning, students are given a learning activities. Learning activities are the main thing in developing students' mindset and knowledge in learning mathematics [7], [26]. In these activities, there are activities that begin with contextual problems in which the problem makes students can develop mathematical concepts.

The final result of Hypothetical Learning Trajectory (HLT) research is in the form of Local Instruction Theory (LIT) that can be generalized according to the characteristics of students. In general, HLT is developed based on student characteristics, material and learning objectives to be achieved. The following is one example of a scheme that we developed for learning mathematics in one of the mathematics material sub-chapters. The following scheme is still a general learning path, and can be developed more specifically with adjustments to student characteristics. The following is the HLT flow scheme on a sub-value or direction ratio in mathematics sub-material.



**Figure 3.** HLT flow scheme

From the figure above, it can be seen that the direction of this learning is horizontal. Each step in which students have activities that are tailored to the goals to be achieved by students. Activities that have been designed can help students to gain new knowledge gradually. If learning goals do not reach, there will be improvements for activities that have not helped students in learning mathematics.

In the developing of designing HLT above, the learning objectives that students will achieve are mathematical material about comparison. Learning begins with the provision of contextual problems that can explore the initial knowledge of students. In making contextual problems, it is important to concern to students' learning characteristics and the extent of students' initial knowledge. Then with that initial knowledge the student know what they have to do to solve that problem. Even though that they do not know how to start to solve that problem, then they will do investigate that problem, and with discussion in small group that students are able to investigate and trial and error in solving these contextual problems. With discussion in small or greater group, the students also are able to get the social knowledge such help friend and attitude in argue. When the students try and investigated the problem, the students also get mathematical concepts gradually to produce a form of formal concept from a comparative or value ratio.

## 4. Conclusion

From the results of the theoretical study above, it can be said that in developing hypothetical learning trajectory (HLT) in mathematics learning there are several things that need to be considered by researchers or educators before composing HLT, namely:

- 1) the characteristics of the material to be taught, so that researchers or educators can think of the learning route that students will through.
- 2) the learning objectives of the material to be achieved by students, so that each route designed will correspond to the goals that students will achieve in shaping mathematical knowledge.
- 3) selection of activities and tasks to be carried out by students in building their knowledge.

We provide suggestions from the results obtained that the results of this study are still in the form of theoretical studies on matters that need to be considered in developing HLT in mathematics learning and the results obtained still need much improvement and further research.

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PAGE 1

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PAGE 2

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PAGE 3

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PAGE 4

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PAGE 5

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PAGE 6

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PAGE 7

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PAGE 8

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